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Swimming and peristaltic pumping between two plane parallel walls

B U Felderhof

Institut für Theoretische Physik A, RWTH Aachen University, Templergraben 55, D-52056 Aachen, Germany

E-mail: ufelder@physik.rwth-aachen.de

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Abstract
Swimming at low Reynolds number in a fluid confined between two plane walls is studied for an infinite plane sheet located midway between the walls and distorted with a transverse propagating wave. It is shown that the flow pattern is closely related to that for peristaltic pumping. The hydrodynamic interaction between two flexible sheets swimming parallel in infinite space is related to the problem of peristaltic pumping in a planar channel with two wavy walls.

1. Introduction

In his path-breaking article on swimming at low Reynolds number Taylor [1] studied a planar sheet propelling itself in an infinite fluid by means of a transverse wave-like distortion. For a propagating wave the surface deformation is not time-reversible and, as a result, the sheet swims with a non-vanishing velocity, as may be calculated in perturbation theory to second order in the wave amplitude. The analysis was based on the creeping flow equations and inertia was neglected. Blake [2] has used the sheet model to study the propulsive effect of bunched cilia on the surface of a microorganism. Taylor's work was extended by Reynolds [3], who included fluid inertia and considered straining of the waving surface. Tuck [4] corrected Reynolds' work by including the convective term in the Navier–Stokes equations.

Reynolds [3] also investigated the influence of one or two nearby walls on the speed of the swimming sheet in the low Reynolds number limit. Further details of the motion in narrow channels were studied by Katz [5]. In section 2 of this paper we present a comprehensive treatment of the swimming at low Reynolds number of a sheet in a planar channel using a method developed in earlier work with Jones [6].

Subsequently we show in section 3 that the same solution for the first-order flow pattern may be used to analyze peristaltic pumping in a planar channel. Early work on peristaltic pumping has been reviewed by Jaffrin and Shapiro [7]. Pumping in planar geometry was studied by Burns and Parkes [8], and by Pozrikidis [9] using a boundary integral method. In a planar channel with two wavy walls there is interference of the wave patterns generated by the individual walls, and the pumping rate and dissipation depend on the phase difference of the two wall distortions. This allows optimization of the pumping rate.

In section 4 we show that the calculation of peristaltic pumping with two wavy walls is closely related to Taylor's analysis [1] of the hydrodynamic interaction of two swimming sheets in infinite space. Taylor considered sheets swimming at the same amplitude and speed, with strokes differing only in phase. We extend his calculation to allow different amplitudes and speeds of the two sheets.

2. Single sheet swimming in a channel

We consider a viscous incompressible fluid of shear viscosity η confined to a planar channel of width $2L$. We choose Cartesian coordinates such that the upper plane wall is at $z = L$ and the lower wall is at $z = -L$. The fluid is assumed to satisfy stick boundary conditions at the walls. In the creeping flow limit the fluid velocity $\boldsymbol{v}(\boldsymbol{r}, t)$ and the pressure $p(\boldsymbol{r}, t)$ satisfy the Stokes equations

$$\eta\nabla^2\boldsymbol{v} - \nabla p = 0, \qquad \nabla\cdot\boldsymbol{v} = 0. \tag{2.1}$$

The time dependence of flow velocity and pressure is caused by the requirement that the flow velocity satisfy the stick boundary condition at a sheet with surface $S(t)$ with prescribed time dependence. The swimming motion of the sheet is defined as the periodic deformation of a planar rest shape S_0 such that for every point $\boldsymbol{s}$ on the plane the position of the corresponding point on the deformed surface $S(t)$:

$$\boldsymbol{R}(t) = \boldsymbol{s} + \boldsymbol{\xi}(\boldsymbol{s}, t), \tag{2.2}$$

is characterized by a displacement vector $\boldsymbol{\xi}(\boldsymbol{s}, t)$, which is periodic in time. The stick boundary condition implies

$$\boldsymbol{v}(\boldsymbol{s} + \boldsymbol{\xi}, t) = \frac{\partial \boldsymbol{\xi}}{\partial t}, \qquad \boldsymbol{s} \in S_0. \tag{2.3}$$

We consider the symmetric case, where S_0 in the rest frame is the xy plane at $z = 0$. The swimming velocity $\boldsymbol{U}$ will be found from the velocity $-\boldsymbol{U}$ of the two walls, as calculated from the prescribed periodic deformation $\boldsymbol{\xi}(t)$.

We construct an approximate perturbative solution to equation (2.1) with boundary condition (2.3) by formal expansion of the velocity field and the pressure in powers of $\boldsymbol{\xi}$:

$$\boldsymbol{v} = \boldsymbol{v}_1 + \boldsymbol{v}_2 + \cdots, \qquad p = p_1 + p_2 + \cdots. \tag{2.4}$$

Both $(\boldsymbol{v}_1, p_1)$ and $(\boldsymbol{v}_2, p_2)$ satisfy equation (2.1). The first-order boundary condition is

$$\boldsymbol{v}_1(\boldsymbol{s}, t) = \frac{\partial \boldsymbol{\xi}}{\partial t}, \qquad \boldsymbol{s} \in S_0. \tag{2.5}$$

Averaging over a period one finds that the averages $\bar{\boldsymbol{v}}_1$ and $\bar{p}_1$ vanish. Hence to first order the swimming velocity $\boldsymbol{U}_1$ vanishes. To second order the swimming velocity $\boldsymbol{U}_2$ is found from the value of the time average $\bar{\boldsymbol{v}}_2(\boldsymbol{r})$ at $z = \pm L$. The averages $\bar{\boldsymbol{v}}_2$ and $\bar{p}_2$ satisfy equation (2.1) with the boundary condition

$$\bar{\boldsymbol{v}}_2(\boldsymbol{s}) = -\overline{(\boldsymbol{\xi} \cdot \nabla)\boldsymbol{v}_1}\big|_{\boldsymbol{r}=\boldsymbol{s}}, \qquad \boldsymbol{s} \in S_0. \tag{2.6}$$

The velocity components $\boldsymbol{v}_1, p_1$ and $\bar{\boldsymbol{v}}_2$ in the upper space $0 < z < L$ are related to those in the lower space $-L < z < 0$ by symmetry.

As an example we consider the transverse displacement

$$\boldsymbol{\xi}(x, t) = A\boldsymbol{e}_z \sin(kx - \omega t), \tag{2.7}$$

with positive wavenumber k and positive frequency ω. By symmetry $v_{1y} = 0$, so that the components v_{1x} and v_{1z} may be derived from a stream function ψ_1 as [10]

$$v_{1x} = \frac{\partial \psi_1}{\partial z}, \qquad v_{1z} = -\frac{\partial \psi_1}{\partial x}. \tag{2.8}$$

We satisfy the boundary condition (2.5) by putting

$$\psi_1(x, z, t) = f(z) \sin(kx - \omega t). \tag{2.9}$$

The pressure p_1 satisfies Laplace's equation, and from the equation for v_{1x} we see that it must take the form

$$p_1(x, z, t) = (P\mathrm{e}^{-k|z|} + Q\mathrm{e}^{k|z|}) \cos(kx - \omega t) \tag{2.10}$$

with coefficients P and Q. Substituting this into the equation for the component v_{1x} and solving, we find that the function $f(z)$ takes the form

$$f(z) = \frac{1}{4\eta k^2}[(B - P(3 + 2k|z|))\mathrm{e}^{-k|z|} + (C + Q(3 - 2k|z|))\mathrm{e}^{k|z|}] \tag{2.11}$$

with coefficients B and C. By using this in the boundary conditions for v_{1x} and v_{1z} at $z = 0$ and L we find for the coefficients

$$\begin{aligned} B &= 2\eta\omega k A\mathrm{e}^{2\kappa} \frac{1 - 2\kappa - 4\kappa^2 - \mathrm{e}^{2\kappa}}{(1 - \mathrm{e}^{2\kappa})^2 - 4\kappa^2\mathrm{e}^{2\kappa}}, \\ C &= 2\eta\omega k A \frac{(1 + 2\kappa - 4\kappa^2)\mathrm{e}^{2\kappa} - 1}{(1 - \mathrm{e}^{2\kappa})^2 - 4\kappa^2\mathrm{e}^{2\kappa}}, \\ P &= 2\eta\omega k A\mathrm{e}^{2\kappa} \frac{1 - 2\kappa - \mathrm{e}^{2\kappa}}{(1 - \mathrm{e}^{2\kappa})^2 - 4\kappa^2\mathrm{e}^{2\kappa}}, \\ Q &= 2\eta\omega k A \frac{1 - (1 + 2\kappa)\mathrm{e}^{2\kappa}}{(1 - \mathrm{e}^{2\kappa})^2 - 4\kappa^2\mathrm{e}^{2\kappa}}, \end{aligned} \tag{2.12}$$

with the abbreviation $\kappa = kL$. The velocity components satisfy the symmetry relations

$$\begin{aligned} v_{1x}(x, z, t) &= -v_{1x}(x, -z, t), \\ v_{1z}(x, z, t) &= v_{1z}(x, -z, t), \end{aligned} \tag{2.13}$$

so that the boundary conditions at $z = -L$ are also satisfied. The mean surface velocity, defined as the right-hand side of equation (2.6), is found to have x component

$$\bar{u}_{xS}(s) = -\overline{(\boldsymbol{\xi} \cdot \nabla)v_{1x}}\big|_{\boldsymbol{r}=\boldsymbol{s}} = U_2, \tag{2.14}$$

with U_2 given by

$$U_2 = \tfrac{1}{2}\omega k A^2 F(\kappa) \tag{2.15}$$

with function

$$F(\kappa) = \frac{\sinh^2 \kappa + \kappa^2}{\sinh^2 \kappa - \kappa^2}. \tag{2.16}$$

The z component of the mean surface velocity vanishes. Hence equation (2.1) is solved by

$$\bar{\boldsymbol{v}}_2(\boldsymbol{r}) = U_2\boldsymbol{e}_x, \qquad \bar{p}_2(\boldsymbol{r}) = 0. \tag{2.17}$$

The swimming velocity of the sheet is, to second order, $\boldsymbol{U}_2 = -U_2\boldsymbol{e}_x$. The sheet swims in the direction opposite to the phase velocity of the surface perturbation. The expression in equation (2.15) was derived also by Reynolds [3] and Katz [5].

The above result implies that in the laboratory frame, where the walls are at rest, the sheet swims with velocity $|U_2|$ in the negative x direction. In the laboratory frame the fluid velocity and the pressure perturbation, when averaged over a period of time, vanish at any point, to second order in the amplitude A. Thus the stick boundary conditions have a pervading influence.

We calculate also the dissipation as a function of the parameters. The rate of dissipation per unit area is

$$D = \eta \int_{-L}^{L} \left[2\left(\frac{\partial v_x}{\partial x}\right)^2 + 2\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2\right] \mathrm{d}z. \tag{2.18}$$

To second order in the amplitude this may be calculated from the first-order flow. Averaging over a period one obtains

$$\bar{D}_2 = \eta A^2\omega^2 k G(\kappa), \qquad G(\kappa) = \frac{\sinh 2\kappa + 2\kappa}{\sinh^2 \kappa - \kappa^2}. \tag{2.19}$$

Alternatively we may follow Taylor [1], and calculate the work done by the fluid pressure against the sheet. The rate of work done per unit area by the fluid in the upper half of the channel is, when averaged over a period,

$$W = \overline{p_1 \frac{\mathrm{d}\xi_z}{\mathrm{d}t}}\bigg|_{z=0+}. \tag{2.20}$$

The viscous stress does not contribute since $\partial v_{1z}/\partial z$ vanishes at the sheet on account of the condition of incompressibility and the vanishing of v_{1x}. We get the same contribution from the lower half of the channel, so that the rate of dissipation is

$$\bar{D}_2 = 2W. \tag{2.21}$$

The relation is confirmed by explicit calculation. The function $G(\kappa)$ tends to 2 as $\kappa \to \infty$, so that in this limit equation (2.19) agrees with the expression for the work calculated by Taylor in the limit $L \to \infty$.

We define the dimensionless efficiency as

$$E_2 = 4\eta\omega \frac{U_2}{\bar{D}_2}. \tag{2.22}$$

The prefactor is chosen such that E_2 equals unity for $L \to \infty$. From equations (2.15) and (2.19) we find

$$E_2 = 2\frac{\sinh^2\kappa + \kappa^2}{\sinh 2\kappa + 2\kappa}. \tag{2.23}$$

In figure 1 we plot the efficiency as a function of κ. The efficiency is maximal at $\kappa_m = 2.065$, corresponding to wavelength $\lambda_m = 3.042L$. At the maximum the efficiency is $E_{2m} = 1.097$. Both functions $F(\kappa)$ and $G(\kappa)$ diverge at small κ, but the efficiency behaves as $E_2 = \kappa + \mathrm{O}(\kappa^3)$.

3. Peristaltic pumping

The same first-order solution can be used to discuss the problem of peristaltic pumping. In this situation the sheet is flexible and can support a running wave, but is kept fixed at the ends, so that there can be no net motion. In each order the flow velocity and pressure again satisfy the creeping flow equations (2.1), but the boundary conditions are different. The solution of the second-order flow problem must be modified accordingly. The second-order flow velocity, averaged over a period of time, must vanish at the walls at $z = \pm L$, but at the mean position of the sheet $z = 0$ there can be a net velocity corresponding to the running wave. As a consequence there is a net force per unit area acting on the sheet, which is compensated by constraints at the ends at $x = \pm\infty$.

Explicitly the second-order averaged flow velocity and pressure are

$$\bar{\boldsymbol{v}}_2(\boldsymbol{r}) = U_2\left(1 - \frac{|z|}{L}\right)\boldsymbol{e}_x, \qquad \bar{p}_2(\boldsymbol{r}) = 0. \tag{3.1}$$

The net current through the upper half of the channel is

$$\boldsymbol{J}_2 = \int_0^L \bar{\boldsymbol{v}}_2(\boldsymbol{r})\,\mathrm{d}z = \tfrac{1}{2}U_2 L\boldsymbol{e}_x, \tag{3.2}$$

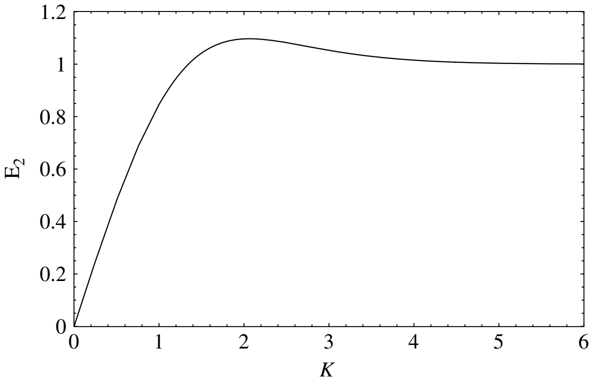

Figure 1. Efficiency E_2 of a single sheet swimming midway in a channel of width $2L$, as given by equation (2.23), as a function of $\kappa = kL$.

and the same current runs through the lower half. The current must be compared with the work done, i.e. the dissipation. The efficiency of the pump is again measured by E_2 given by equation (2.23).

It is natural to consider also a pump with two wavy walls consisting of two sheets at $z = 0$ and L. In this case we consider only the space $0 < z < L$. We assume again that the lower sheet is distorted by the running wave given by equation (2.7). The upper sheet is distorted similarly with a running wave

$$\hat{\boldsymbol{\xi}}(x,t) = \hat{A}\boldsymbol{e}_z \sin(kx - \omega t - \varphi), \tag{3.3}$$

with phase shift φ. The first-order flow is derived from the stream function

$$\psi_1(x,z,t) = f(z)\sin(kx - \omega t) + \hat{f}(z)\sin(kx - \omega t - \varphi). \tag{3.4}$$

We write the function $\hat{f}(z)$ in the form

$$\hat{f}(z) = \frac{1}{4\eta k^2}[(\hat{B} - \hat{P}(3 + 2k(L-z)))\mathrm{e}^{-k(L-z)} + (\hat{C} + \hat{Q}(3 - 2k(L-z)))\mathrm{e}^{k(L-z)}], \tag{3.5}$$

similar to equation (2.11). Then the coefficients $\hat{B}$, $\hat{C}$, $\hat{P}$, $\hat{Q}$ take the same form as in equation (2.12) with A replaced by $\hat{A}$. The mean surface velocity at the lower sheet takes the form of equation (2.14) with U_2 replaced by U_2' with

$$U_2' = \tfrac{1}{2}\omega k A[AF(\kappa) + \hat{A}H(\kappa)\cos\varphi], \tag{3.6}$$

with $H(\kappa)$ given by

$$H(\kappa) = \frac{2\kappa\sinh\kappa}{\sinh^2\kappa - \kappa^2}. \tag{3.7}$$

The mean surface velocity at the upper sheet takes the same form with U_2 replaced by $\hat{U}_2'$, where

$$\hat{U}_2' = \tfrac{1}{2}\omega k\hat{A}[\hat{A}F(\kappa) + AH(\kappa)\cos\varphi]. \tag{3.8}$$

The second-order average flow velocity is

$$\bar{\boldsymbol{v}}_2(\boldsymbol{r}) = \left[U_2'\left(1 - \frac{z}{L}\right) + \hat{U}_2'\frac{z}{L}\right]\boldsymbol{e}_x, \tag{3.9}$$

corresponding to the net current

$$\boldsymbol{J}_2 = \int_0^L \overline{\boldsymbol{v}}_2(\boldsymbol{r})\,\mathrm{d}z = \tfrac{1}{2}(U_2' + \hat{U}_2')L\boldsymbol{e}_x. \tag{3.10}$$

The second-order average dissipation is

$$\bar{D}_{2\mathrm{p}} = \tfrac{1}{2}\eta\omega^2 k[A^2 G(\kappa) - 4A\hat{A}K(\kappa)\cos\varphi + \hat{A}^2 G(\kappa)], \tag{3.11}$$

with interference factor

$$K(\kappa) = \frac{\kappa\cosh\kappa + \sinh\kappa}{\sinh^2\kappa - \kappa^2}. \tag{3.12}$$

We define the efficiency of the pump as

$$E_{2\mathrm{p}} = 2\eta\omega\frac{U_2' + \hat{U}_2'}{\bar{D}_{2\mathrm{p}}}, \tag{3.13}$$

and consider in particular the case of equal amplitudes $A = \hat{A}$. Then $U_2' = \hat{U}_2'$, so that the second-order flow on time average is uniform across the channel. The efficiency becomes

$$E_{2\mathrm{p}} = 2\frac{F(\kappa) + H(\kappa)\cos\varphi}{G(\kappa) - 2K(\kappa)\cos\varphi}. \tag{3.14}$$

This tends to unity at large κ and shows a maximum with value $E_{2\mathrm{pm}} > 1$ provided $0 < |\varphi| < 1.6345$. For example, for $\varphi = \pi/4$ the maximum is at $\kappa_\mathrm{m} = 1.227$ and takes the value $E_{2\mathrm{pm}} = 4.727$. For smaller values of φ the value of κ_m decreases and $E_{2\mathrm{pm}}$ increases. As $\varphi \to 0$ the value κ_M tends to 0 and $E_{2\mathrm{pm}}$ tends to ∞. In figure 2 we show $E_{2\mathrm{p}}$ at $\varphi = \pi/4$ as a function of κ. Both the speed and the dissipation diverge at small κ, but the efficiency behaves as

$$E_{2\mathrm{p}} = \kappa\cot^2\frac{\varphi}{2} + \mathrm{O}(\kappa^3) \qquad \text{as } \kappa \to 0. \tag{3.15}$$

The slope diverges as $\varphi \to 0$.

4. Two swimming sheets

The calculation of section 3 of a peristaltic pump consisting of a channel bounded by two flexible sheets may be used to discuss the situation of two sheets swimming parallel in infinite space. The situation was studied by Taylor [1] in connection with the hydrodynamic interaction of spermatozoa. He considered the case of two sheets swimming at the same speed, with strokes differing only by a phase angle. We consider the more general case where amplitudes and speeds may also differ. Physically one may think of two swimmers who tune their Doppler-shifted frequencies such that their wave patterns are in resonance and show constructive interference. In the small-amplitude limit the Doppler shifts vanish, so that one may consider a single frequency.

We consider a lower sheet swimming in infinite fluid with distortion in its rest frame given by equation (2.7) and an upper sheet separated by distance L swimming with distortion in its rest frame given by equation (3.3). To first order in the amplitudes the flow is a linear superposition of the two flows of the individual swimmers. As a consequence the lower sheet

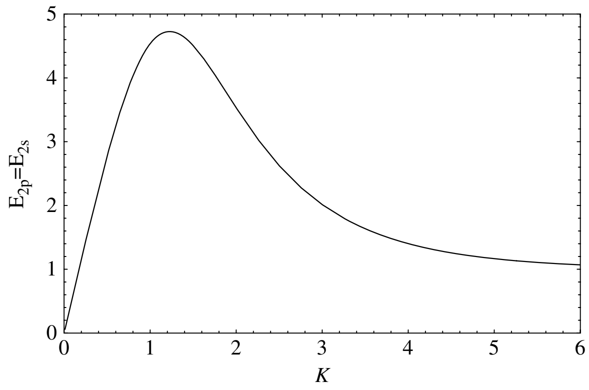

Figure 2. Efficiency $E_{2\mathrm{p}}$ of a peristaltic pump of width L, as given by equation (3.14), as a function of $\kappa = kL$ for phase difference $\varphi = \pi/4$. The same plot gives the efficiency $E_{2\mathrm{s}}$ of two sheets swimming at a distance L in infinite space with strokes of equal wavenumber and amplitude with phase difference $\varphi = \pi/4$.

swims with velocity $\boldsymbol{U}_{21} = -U_2'\boldsymbol{e}_x$ in the frame where the fluid is at rest at infinity and the upper sheet swims with velocity $\boldsymbol{U}_{22} = -\hat{U}_2'\boldsymbol{e}_x$, where U_2' and $\hat{U}_2'$ are given by equations (3.6) and (3.8). The mean second-order flow velocity $\bar{\boldsymbol{v}}_2(\boldsymbol{r})$ vanishes identically.

It is not necessary to calculate the dissipation in the upper space $z > L$ and in the lower space $z < 0$ explicitly, since we can use Taylor's argument which gives equation (2.21) for a single swimmer. Thus the total mean dissipation for the two swimming sheets to second order in the two amplitudes is

$$\bar{D}_{2\mathrm{s}} = 2\bar{D}_{2\mathrm{p}}, \tag{4.1}$$

and can be found from equation (3.11). With the definitions

$$\alpha = \frac{1}{2}G(\kappa) - K(\kappa) = \frac{2\sinh^2(\kappa/2)}{\sinh\kappa + \kappa},$$
$$\beta = \frac{1}{2}G(\kappa) + K(\kappa) = \frac{2\cosh^2(\kappa/2)}{\sinh\kappa - \kappa}, \tag{4.2}$$

this can be cast in the form

$$\bar{D}_{2\mathrm{s}} = \eta\omega^2 k[(A^2 + \hat{A}^2)(\alpha + \beta) + 2A\hat{A}(\alpha - \beta)\cos\varphi]. \tag{4.3}$$

For equal amplitudes $A = \hat{A}$ and with $\varphi = 2\phi$ this reduces to

$$\bar{D}_{2\mathrm{s}} = 4\eta\omega^2 kA^2[\alpha\cos^2\phi + \beta\sin^2\phi], \tag{4.4}$$

which is the form derived by Taylor. He did not calculate the velocity of the two swimmers. For $A = \hat{A}$ we define the efficiency of the swimming pair as

$$E_{2\mathrm{s}} = 8\eta\omega\frac{U_2'}{\bar{D}_{2\mathrm{s}}}. \tag{4.5}$$

This takes the value $E_{2\mathrm{s}} = E_{2\mathrm{p}}$, with $E_{2\mathrm{p}}$ given by equation (3.14). For given phase difference $\varphi < \pi/2$ the efficiency can be optimized by varying the wavenumber k, as discussed in section 2.

The above calculation relies on the fact that, to first order in the amplitude, one cannot distinguish between the laboratory frame and the rest frame of either sheet. Hence the boundary condition for the second-order flow and the dissipation can be calculated from the first-order flow in the common laboratory frame in which the fluid is at rest at infinity. The time-averaged second-order surface velocity in the rest frame of either sheet, which determines the boundary condition, is constant and directed along the sheet. The corresponding problem for the mean second-order flow velocity in the laboratory frame, with the condition that neither sheet exert a net force on the fluid, has a trivial solution.

5. Discussion

We have shown that the problems of swimming and peristaltic pumping at low Reynolds number in planar geometry may be solved from a unified point of view. The solution of the creeping flow equations to first order in the wave amplitudes is identical for the two problems. The difference arises to second order in the amplitudes. We have considered a transverse wave and symmetric geometry, but clearly the principle holds more generally. Different types of waves and different geometries may be considered.

For both problems it would be of interest to include the effects of inertia. This would imply a generalization of Tuck's solution [4] for the swimming of a single sheet.

There is an extensive literature on the swimming of bodies of finite size at low Reynolds number [11, 6, 12–15]. It is usually assumed that the swimmer moves in infinite fluid. The example treated here shows that confinement by walls can have an important effect on the swimming velocity, and can improve the efficiency.

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